

# OFDM based on FFT decoder with non-integer argument

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**Abstract**— Due to its efficient use of the spectral band and also to its immunity to selective fading, Orthogonal Frequency Division Multiplexing (OFDM) modulation was adopted by many of the wireless technologies of today. However resilient this modulation technique is to Inter Symbol Interference (ISI) or Inter Carrier Interference (ICI), the errors determined by this interference mechanism are significant. The present paper proposes the development of a new OFDM decoding algorithm which can demodulate data in severe ISI or ICI conditions. It is based on use of oversampling in frequency technique, performed by a Discrete Fourier Transform (DFT) with non-integer argument. The obtained results are reported against the standard ones.

**Keywords**— OFDM, oversampling in frequency, interference errors

## I. INTRODUCTION

OFDM is a modulation method with superior performances and benefits compared to traditional ones, in terms of spectrum efficiency, good resilience to interference, immunity to selective fading and channel equalization. Therefore, it can be found in many Wi-Fi networks, WiMAX, 4G mobile communication Long Term Evolution (LTE), digital subscriber line (DSL), majority of power-line communications applications (PLC), ADSL and VDSL internet access modems, military applications and satellite communications [1]-[4]. However, significant transmission errors, caused by interference mechanisms have been reported [5]. Since the subject of this paper is to propose a new method to reduce them, the most important one, ICI, is presented below.

Consider a symbol of  $N=100$  points (samples) having a non-integer number of cycles (4.5 cycles), thus a symbol affected by ICI. In this case, not a single bin is synchronized with the maximum power (Fig 1). For example, the calculation of the fourth bin is achieved by multiplying the time sequence with a  $\sin()$  and  $\cos()$  signals of 4 cycles. After multiplication, the signals are averaged and summed, yielding bin's value. Only the  $\cos()$  multiplication is presented graphically because, for the considered case, the  $\sin()$  multiplication leads to a zero average. Because ICI makes the time sequence, subject to Fourier Transform, to contain a non-integer number of periods, the amplitude and phase of a subcarrier (evaluated through a

complex bin) will be inaccurate. Numerous researches were conducted [6] in order to reduce the ICI errors. Also, a detailed

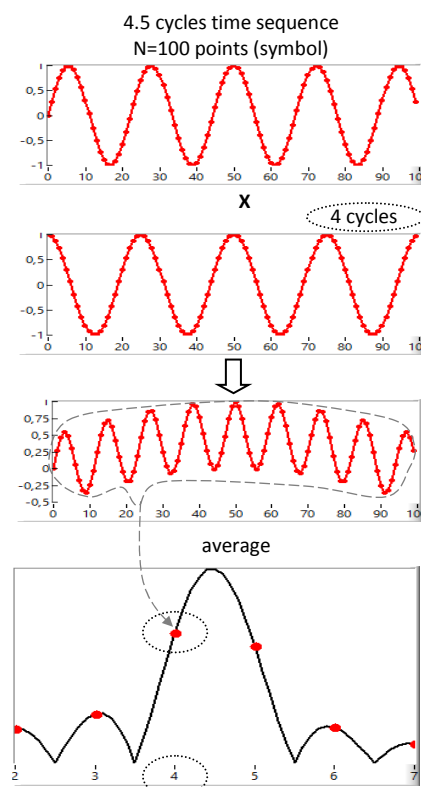


Fig. 1. Obtain a frequency point

presentation of existing methods used to reduce ISI errors can be found in [5]. A class of them is using the oversampling in frequency techniques, consisting in a fractional division of frequency domain. Its main idea is presented below.

If the time sequence consists of  $N$  points then, by oversampling,  $M$  points are obtained in frequency domain, with  $M > N$ . These  $M$  points can be obtained in  $M \cdot N$  operations through Z-Transform [7],  $(M+N) \log(M+N)$  operations through Chirp Z [5] or  $M \log M$  operation if the time sequence is extended with  $(M-N)$  zeroes and FFT is calculated [8].

This paper propose a new oversampling in frequency technique to be used, technique that is computationally efficient, as making use of  $M\log(N)$  operations.

## II. OFDM BASED ON FFT DECODER WITH NON-INTEGGER ARGUMENT

The zero-padded oversampling in frequency method was reported as being used in OFDM decoders for radiofrequency [9]-[10] and optical [11] transmissions. It consists in the addition of  $(M-N)$  zeroes to the time sequence, the FFT evaluation yielding  $M$  frequency points. It is preferred over all other oversampling methods because of its low computational effort.

Considering the example from Fig. 1, let's suppose that we want to better identify the value of the maximum power and for this we use oversample in frequency. If we want to double the number of frequency points, we will double the number of points in time domain. Thus, the time sequence of 4.5 cycles and  $N=100$  is increased by  $M-N=100$  zeroes. As seen in figure 2, doubling the length of the time sequence leads to widening the frequency domain, which conducts to halving the  $\Delta f$  bins. In these conditions the 9<sup>th</sup> bin is obtained by multiplying the sequence with  $\sin()$  and  $\cos()$  signals containing an integer number of cycles (9 cycles). These multiplications lead to a nonzero signal only over the domain where the time sequence wasn't extended with zeroes.

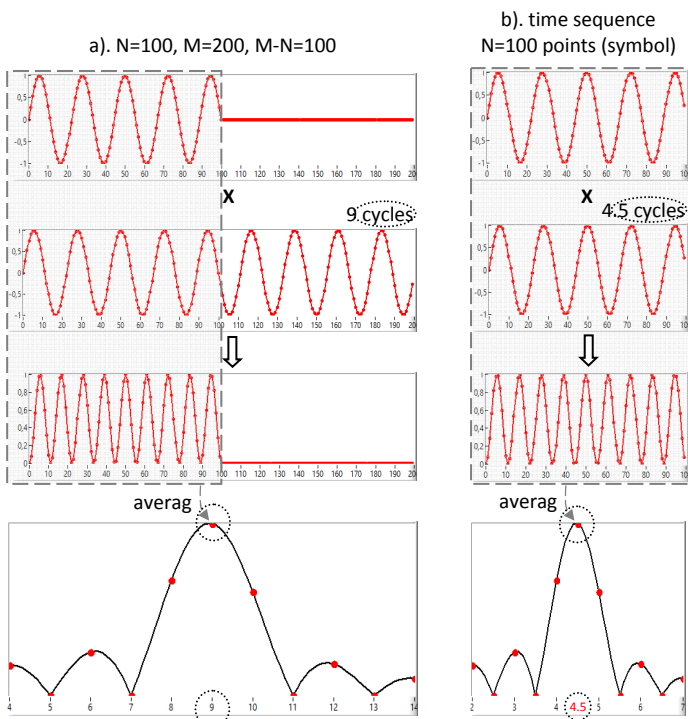


Fig. 2. a - Doubling bins resolution through zero-padding and obtaining the value of a frequency point, b - Doubling the bins resolution using fractional argument

All multiplications made between the zeroes of the time sequence and  $\sin()$  and  $\cos()$  signals have no contribution to the average of the signal obtained from multiplying. The zero

padding method simply increase the number of values which can be taken by the number of cycles of  $\sin()$  and  $\cos()$  signals.

An alternate oversampling in frequency method, yielding similar results with much lower computational effort was reported in [12] and some components and applications in [13]-[16].

The active part (non-zero) of the zero padding algorithms, in Fig. 2a stretches over the time sequence not extended with zeroes in Fig. 2b. But at the same time it can be seen that  $\sin()$  and  $\cos()$  functions doesn't have an integer number of periods but only 4.5 cycles. The marked (active) area from Fig.2a is extracted and used to implement a method to calculate the DFT with non-integer argument, which develops the same frequency spectrum as the zero-padding.

The implementation of DFT with non-integer argument into a FFT with non-integer argument allows us to obtain a frequency spectrum with  $M$  frequency points, in  $M\log N$  number of operations.

## III. EXPERIMENTAL SETUP AND RESULTS

Tests have been made with a virtual instrument developed by using LABVIEW RF Communications/Modulations Toolkit. It comprises two main modules, the transmitter and the receiver one. The transmitter module provides data in standard OFDM format but also allow different kind of disturbances to be added (IQ Impairments, sample offset, fading profiles, AWGN and phase noise).

The receiver module consists in two parallel processing units. The decoder of the first unit make use of the standard FFT algorithm while the decoder of the second one use an implementation of a FFT with non-integer argument.

The obtained results have been evaluated in terms of Bit Error Rate (BER).

Test conditions:

- bits transmitted from generator to receiver for BER calculation: 75750;
- BER calculation: number of erroneous bits / total number of transmitted bits;
- map type: 4QAM
- IQ rate: 5.16 MHz

Test parameters:

- AWGN;
- frequency offset;
- fading Profile.

Type of test: application of one disturbance at a time.

Various values of the oversampling factor  $F$  (defined as the ratio between the number of points in frequency sequence and the number of points in time sequence) have been used and some of the obtained results are presented bellow.

In all of the following figures the black plots are representing the results obtained by using the standard decoder while the red plots are representing the results obtained by using the proposed one.

### A. Immunity tests for an AWGN channel

For the analysis of a noisy channel, we evaluated BER as a function of the normalized carrier-to-noise ratio (or energy per bit to noise power spectral density ratio) hereafter denoted  $E_b/N_0$ . Results obtained for two different values of the oversampling factor ( $F=10$  and  $F=100$ ) are presented in Fig.3.

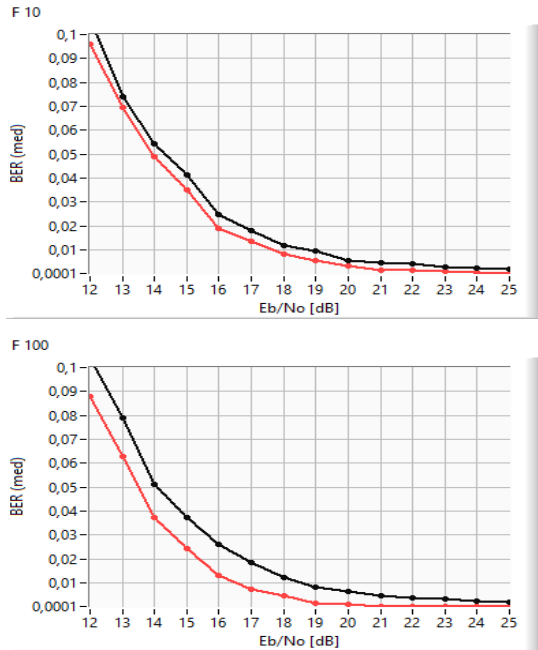


Fig. 3. Results obtained for an AWGN channel

Significant better values for BER have been obtained by using the proposed decoder and, as expected, once the oversampling factor increase, the performances are increasing.

### B. Frequency offset

The behaviour of the proposed decoder has been analyzed for four different values of frequency offset (200, 300, 400, and 500 Hz). In all of these cases the decoder emphasizes similar improved performances as compared to the standard one. Fig. 4 presents the results obtained for a frequency offset of 200 Hz and an oversampling factor  $F=100$ .

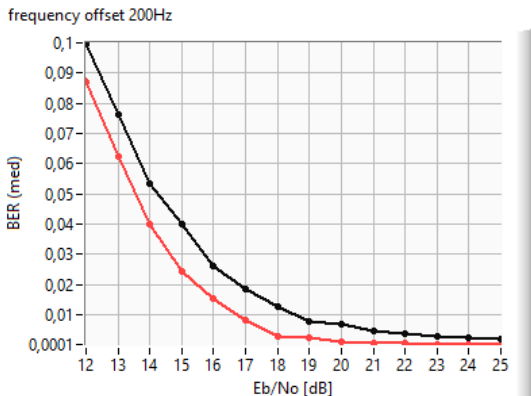


Fig.4. Results obtained for frequency

### C. Immunity testing for a fading channel

In estimating the behavior of the proposed decoder for a fading channel we used the Rician model of the channel, assuming the presence of a dominant transmission path. In addition, we assumed a 2 GHz Doppler spread of the channel associated with the non stationary status of the users (or reflectors).

For several values of the oversampling factor  $F$ , we determined BER value against Rician parameter  $K$ , the last one denoting the relative strength of the direct line of sight path component. Some of the results are depicted in Fig.5. The upper graph is presenting the results obtained for  $F=100$  while the lower one for  $F=10$ .

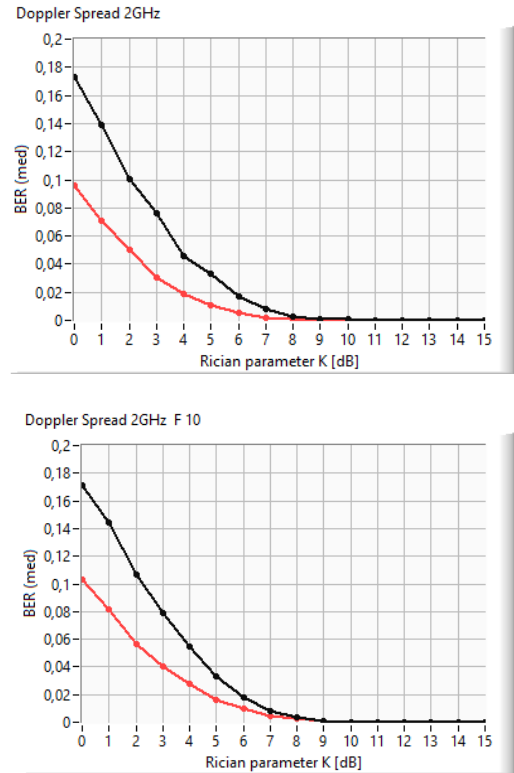


Fig.5. Results obtained for fading channels

It is obviously that both situations exhibit a very good behaviour of the proposed decoder in terms of BER, with minor differences between  $F=10$  and  $F=100$ .

### CONCLUSIONS

A straightforward FFT implementation of the DFT with non-integer argument was used to simulate the core decoder module of an OFDM receiver. The obtained results, in terms of Bit Error Rate, have been presented against those obtained with the standard decoder. A very good behavior was observed for both noisy and fading channels, and also for frequency offset..

### ACKNOWLEDGMENT

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